USM-TH-80

Electric Screening Mass of the Gluon with Gluon Condensate at Finite Temperature

Iván Schmidt[1] and Jian-Jun Yang [1,2]

[1] *Departamento de Física, Universidad Técnica Federico Santa Maria, Casilla 110-V, Valparaíso, Chile*

[2] *Department of Physics, Nanjing Normal University, Nanjing 210097, P. R. China*

Abstract

The electric screening mass of the gluon at finite temperature is estimated by considering the gluon condensate above the critical temperature. We find that the thermal gluons acquire an electric mass of order T due to the gluon condensate.

PACS number(s): 12.38.Mh; 12.38.Lg; 52.25.Kn

1. Introduction

The color screening effect is one of main features of the high temperature quark-gluon plasma(QGP). The value of the electric screening mass ($m_e \sim g(T)T$), which gives rise to the Debye screening of the heavy quark potential, has been known in perturbative theory for quite some time [1,2]. Although much progress in perturbative calculations [3] has been achieved by the newly developed techniques of resumed perturbation theory, it seems that non-perturbative effects still dominate in the temperature regime attainable in the near future heavy ion experiments. Especially in the regime right above the critical temperature, nonperturbative effects are supposed to be important.

Quark and gluon condensates, which describe the nonperturbative effects of the QCD ground state, have been extensively used in QCD sum rules [4] in order to study hadron properties at zero and finite temperature. Recently, the quark propagator at finite temperature including the gluon condensate was calculated by Schäfer and Thoma [5] and also the contribution of the gluon condensate to the gluon propagator at zero temperature has been extensively studied [6–13]. Here we will extend these investigations to the study of the self energy of the gluon at finite temperature by using the thermal gluon propagator. The main purpose of this paper aims at estimating the gluon contribution to the electric screening mass of the gluon at finite temperature.

2. Gluon Condensate Contribution to Electric Screening Mass of the Gluon at Finite Temperature

The lowest order contribution of the gluon, ghost and quark condensates to the gluon propagator is depicted by diagrams in Fig. 1. The presence of the quark condensate comes from chiral symmetry breaking, so its contributions in Fig. 1(e-f) vanish due to chiral symmetry restoration above the phase transition. The gluon condensate above the critical temperature which is associated with the breaking of scale invariance has been measured on the lattice recently [14]. We will focus on the gluon condensate contribution to the self energy of the gluon. In principle, in order to preserve the Slavnov-Taylor identity(STI) [15], the

ghost condensate contributions in Fig.1(b-c) should be taken into account to make sure that the gluon self energy including the gluon condensate is transverse, which makes the problem complicated since the expression for the gluon self energy contains unknown condensates. However, as pointed by Lavelle [10], the gluon condensate is always the main contribution to the nonperturbative gluon at $T = 0$. We assume that this still holds in the case $T \neq 0$. To avoid unnecessary complications, here we only analyze the gluon condensate contribution to the self energy of the gluon at small momenta and extract the electric screening mass of the gluon due to nonperturbative effects above the phase transition.

The real part of the gluon self-energy in the one-loop approximation is determined by the 1-1 component of the Feynman propagator in thermo-field dynamics (TFD). Similar to the perturbative case, using the Feynman rules of thermo-field dynamics [16,17] and adopting the imaginary time formalism, the self-energy of the gluon with the gluon condensate contributions shown in Fig. 1, is

$$\begin{aligned}\Pi_{\mu\nu}(p_0,\vec{p}) = & -\frac{iN_c}{(N_c^2-1)}4\pi\alpha_s iT\sum_{k_0=2\pi inT}\int\frac{d^3k}{(2\pi)^3}D^{(\mathrm{NP})}_{\lambda\lambda'}(k_0,\vec{k}) \\ & \times[(2p-k)_\lambda g_{\mu\rho}+(-p+2k)_\mu g_{\rho\lambda}+(-k-p)_\rho g_{\lambda\mu}] \\ & \times[(2p-k)_{\lambda'} g_{\sigma\nu}+(-p-k)_\sigma g_{\nu\lambda'}+(2k-p)_\nu g_{\lambda'\sigma}] \\ & \times[-\frac{g^{\rho\sigma}}{(p-k)^2}+\frac{(1-\xi)(p-k)^\rho(p-k)^\sigma}{(p-k)^4}] \end{aligned} \tag{1}$$

where at finite temperature the zeroth components of momentum 4-vectors take on discrete values, namely, $k_0 = 2\pi inT$ with integer n. This is a direct consequence of Fourier analysis in the imaginary time formation of Matsubara [18]. The lowest Matsubara mode with $n = 0$ should be a good approximation as long as p is not much larger than the critical temperature T_c. At zero temperature, the transversality and Lorentz-invariance of the gluon polarization tensor requires it to have the form $\Pi_{\mu\nu}(k) = \Pi(k^2)(g_{\mu\nu} - k_\mu k_\nu/k^2)$. At finite temperature, Lorentz-invariance is lost and the polarization operator presents a combination of transverse and longitudinal tensor structures. Therefore, the nonperturbative gluon propagator $D^{(\mathrm{NP})}_{\lambda\lambda'} = D^{full}_{\lambda\lambda'} - D^{pert}_{\lambda\lambda'}$, which contains the gluon condensate, is generally assumed to have the form [19]

$$D^{(\mathrm{NP})}_{\lambda\lambda'}(k_0,\vec{k}) = D_L(k_0,\vec{k})P^L_{\lambda\lambda'} + D_T(k_0,\vec{k})P^T_{\lambda\lambda'}, \tag{2}$$

where $D_{T,L}$ are the transverse and longitudinal parts of the nonperturbative gluon propagator at finite temperature, and the bare gluon propagator has been subtracted since we are not interested in perturbative corrections to the gluon self energy [5]. In (2), the longitudinal and transverse projectors can be written as

$$P^L_{\lambda\lambda'} = \frac{k_\lambda k_{\lambda'}}{k^2} - g_{\lambda\lambda'} - P^T_{\lambda\lambda'} \tag{3}$$

$$P^T_{\lambda 0} = 0, P^T_{ij} = \delta_{ij} - \frac{k_i k_j}{k^2} \tag{4}$$

Using the same method as in Ref. [5], we expand the gluon propagator in (1) for small loop momenta k and keep only terms which are bilinear in k to relate the gluon condensate to moments of the gluon propagator, and we obtain

$$\Pi_{00}(p_0=0,\vec{p}) = \frac{4\pi\alpha_s N_c}{(N_c^2-1)} T \int \frac{d^3\vec{k}}{(2\pi)^3} (\frac{8}{9} D_T - \frac{136}{45} D_L) \frac{\vec{k}^2}{\vec{p}^2}. \tag{5}$$

The moments of the longitudinal and transverse gluon propagators are related to the chromoelectric and chromomagnetic condensates via [5]

$$\langle \vec{E}^2 \rangle_T = 8T \int \frac{d^3\vec{k}}{(2\pi)^3} \vec{k}^2 D_L(0,\vec{k}), \tag{6}$$

$$\langle \vec{B}^2 \rangle_T = -16T \int \frac{d^3\vec{k}}{(2\pi)^3} \vec{k}^2 D_T(0,\vec{k}). \tag{7}$$

Therefore, $\Pi_{00}(p_0=0,\vec{p})$ can be re-written as

$$\Pi_{00}(p_0=0,\vec{p}) = \frac{4\pi\alpha_s N_c}{(N_c^2-1)} [-\frac{1}{18}\langle \vec{B}^2 \rangle_T - \frac{17}{45}\langle \vec{E}^2 \rangle_T] \frac{1}{\vec{p}^2}. \tag{8}$$

Furthermore, from the expectation values of the space and timelike plaquettes $\Delta_{\sigma,\tau}$ of lattice QCD [14], the chromoelectric and chromomagnetic condensates can be extracted as

$$\frac{\alpha_s}{\pi}\langle \vec{E}^2 \rangle_T = \frac{4}{11}\Delta_\tau T^4 - \frac{2}{11}\langle \vec{G}^2 \rangle_{T=0}, \tag{9}$$

$$\frac{\alpha_s}{\pi}\langle\vec{B}^2\rangle_T = -\frac{4}{11}\Delta_\sigma T^4 + \frac{2}{11}\langle\vec{G}^2\rangle_{T=0}, \tag{10}$$

Hence, we obtain,

$$\Pi_{00}(p_0=0,\vec{p}) = \frac{4N_c\pi^2}{(N_c^2-1)\vec{p}^2}[\frac{2}{99}(\Delta_\sigma - \frac{34}{5}\Delta_\tau)T^4 + \frac{29}{495}\langle G^2\rangle_{T=0}] \tag{11}$$

where the gluon condensate at zero temperature is taken as [14]

$$\langle G^2\rangle_{T=0} = (2.5\pm 1.0)T_c^4 \tag{12}$$

The critical temperature is taken as $T_c = 260MeV$ [14] in the following calculations.

The electric screening mass is related to the low momentum behavior of the gluon polarization tensor, $\Pi_{\mu\nu}(p_0,\vec{p})$. It is generally defined as the zero momentum limit ($|\vec{p}|\to 0$) in the static sector ($p_0=0$) of $\Pi_{00}(p_0,\vec{p})$ [19], i.e.,

$$m_e^2 = \Pi_{00}(0,|\vec{p}|\to 0) \tag{13}$$

However, the above definition cannot be correct since beyond leading order in the coupling it is gauge dependent in non-Abelian theories [20,21]. A better way is to define the electric screening mass as the position of the pole of the propagator at spacelike momentum [20,21]

$$p^2 + \Pi_{00}(0,p^2=-m_e^2) = 0, \tag{14}$$

where $p=|\vec{p}|$. Using this definition, the leading perturbative contribution to the electric screening mass reads [1],

$$m_e^2(T) = (\frac{N_c}{3}+\frac{N_f}{6})g^2(T)T^2. \tag{15}$$

Using the lattice results for the plaquette expectation values [14], the electric screening mass can be solved from Eq. (14). The numerical results are shown in Fig. 2. From Fig. 2, one can see that the screening mass due to the gluon condensate is almost proportional to T when $T>1.3T_c$. In addition, the electric screening mass is not sensitive to the value of the zero temperature gluon condensate.

3. Summary and Discussion

To sum up, we investigated the gluon condensate contribution to the electric screening mass of the gluon at finite temperature. In the plasma, the gluons acquire an electric screening mass which is approximately proportional to T. This screening mass comes mainly from the thermal gluon condensate and is not sensitive to the value of the zero temperature gluon condensate. The electric mass gives rise to the Debye screening of the heavy quark potential. The magnitude of m_e influences strongly the existence or non-existence of charmonium in the high temperature phase according the following behaviour of the potential $V(r)$ between gauge invariant sources [22]:

$$V(r) \propto \frac{\mathrm{e}^{-2\mathrm{m_e r}}}{r^2}. \tag{16}$$

Therefore, the gluon screening mass of order T due to nonperturbative effects is supposed to influence the dissociation of charmonium in the QGP.

In principle, this work can be extended to study the screening behavior of the gluon magnetic sector. It is well known that the absence of static magnetic screening in the hard thermal loop resummed propagator leads to infrared singularities in perturbative calculations, i.e. the zero value of the magnetic mass-squared results in the divergence of the expansion of the thermodynamic potential. However, a static magnetic screening always vanishes in perturbative calculations. A magnetic mass of the gluon might appear in the nonperturbative gluon propagator. Unfortunately, in order to study the transversality of the gluon self energy, the STI should be considered exactly. Hence the ghost condensate and higher order condensates at finite temperature have to be included.

ACKNOWLEDGEMENTS

We would like to thank Drs. M. Lavelle and M.H. Thoma for discussions. This work was supported in part by Fondecyt (Chile) postdoctoral fellowship 3990048, by Fondecyt (Chile) grant 1990806 and by a Cátedra Presidencial (Chile), and also by Natural Science Foundation of China grant 19875024.

REFERENCES

[1] E.V. Shuryak, Sov. Phys. JETP **47**, 212 (1978) .

[2] O.K. Kalashnikov, Fortschr. Phys. **32**, 525 (1984).

[3] E. Braaten and R. Pisarski, Nucl. Phys. **B337**, 569 (1990).

[4] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. **B147**, 385, 448, 519 (1979); J. Schwinger, *Quantum Chromodynamics,* Chap. IV., Eds. W. Beiglbock et al., (Spring-Velag, New York, 1983).

[5] A. Sch*ä*fer and M. H. Thoma, Phys. Lett. **B 451**, 195 (1999).

[6] J.J. Yang, H.Q. Shen, G.L. Li, T. Huang and P.N. Shen Nucl. Phys. **A640**, 457 (1998).

[7] M.J. Lavelle and M. Schaden, Phys. Lett. B **208**, 297 (1988).

[8] M. Lavelle and M. Schaden, Phys. Lett. B **217**, 551 (1989).

[9] M. Lavelle and M. Schaden, Phys. Lett. B **246**, 487 (1990).

[10] M. Lavelle, Phys. Rev. D **44**, R26 (1991).

[11] J. Ahlbach, M. Lavelle, M. Schaden and A. Streibl, Phys. Lett. B **275**, 124 (1992).

[12] E. Bagan and T. G. Steele, Phys. Lett. B **219**, 497 (1989); E. Bagan, M. R. Ahmady, V. Elias and T. G. Steele, Z. Phys. C **61**, 157 (1994).

[13] V. Elias, T. G. Steele and M. D. Scadron, Phys. Rev. D **25**, 1584 (1988).

[14] G. Boyd et al., Nucl. Phys. **B469**, 419 (1996).

[15] J. C. Taylor, Nucl. Phys. **B33**, 436 (1971); A. A. Slavnov, Theor. Math. Phys. **10**, 99 (1972).

[16] N.P. Landsman and Ch. G. Van Weert, Phys. Rep. **145**, 141 (1987).

[17] R.L. Kobes and G.W. Semenoff, Nucl. Phys. **B 260**, 714 (1985).

[18] A.L. Fetter and J.D. Walecka, Quantum Theory of Many-Particle Systems. McGraw-Hill. San Francisco, 1971.

[19] J.I. Kapusta, Finite Temperature Field Theory, Cambridge University press, Cambridge 1989.

[20] A.K. Rebhan, Nucl. Phys. **B430**, 319 (1994).

[21] A.K. Rebhan, Phys. Rev. **D48**, R3967 (1993).

[22] M.L. Bellac, Thermal Field Theory, Cambridge University Press, Cambridge 1996.

FIGURES

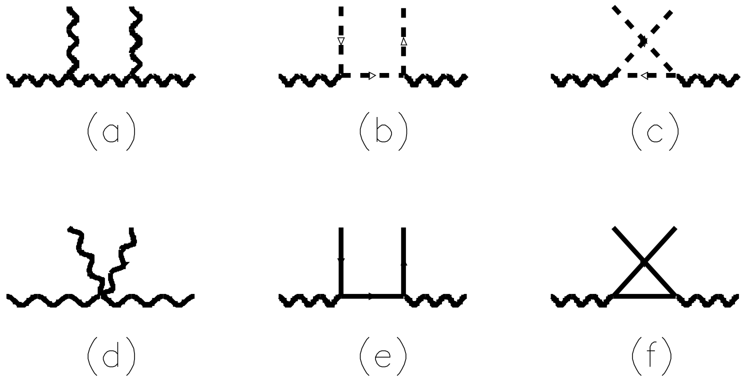

FIG. 1. The Feynman diagrams for the contributions of the nonperturbative corrections to the gluon propagator with the lowest dimensional gluon, ghost and quark condensates.

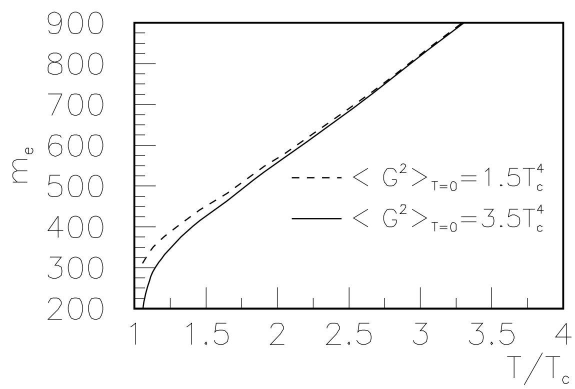

FIG. 2. The electric screening mass m_e versus T/T_c